Some Hadronic Properties with Light Front Holography

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Outline

Introduction

Applicability to QCD.

- N=4 SYM is different to QCD, but we can argue that in some situations both are closer. Ej: Heavy Ion Collisions.
- Gauge / Gravity ideas can be expanded in several directions. This give us a possibility to get a field theory similar to QCD with gravity dual.
- You can use Gauge / Gravity as a nice frame to built phenomenological models with extra dimensions that reproduce some QCD facts.

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- Top - Down aproach.
  You start from a string theory on $AdS_{d+1}xC$, and try to get at low energies a theory similar to QCD in the border.
- Bottom - Up approach.
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⋆ **Dictionary.**
This tell us how are related elements involved in both sides of Gauge / Gravity duality.

Table: Summary of dictionary considered here.

QCD (4d)	Gravity (5d)
Operator ($\mathcal{O}$)	Normalizable Modes (Φ)
Hadron Mass (M)	Eigenvalues of Φ
Twist Dimension ([$\mathcal{O}$] - S)	Conformal Dimension (Δ)
Wave Function	Normalizable Modes (Φ) [1]

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Mesonic Phenomenology. [2]

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T. Branz, T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. V, Phys. Rev. D **82**, 074022 (2010) (arXiv:1008.0268).

$$Light\ Front: \quad F(q^2) = 2\pi \int_0^1 dx \frac{(1-x)}{x} \int d\zeta \zeta J_0(\zeta q \sqrt{\frac{1-x}{x}}) \frac{|\psi_{q_1 \bar{q}_2}|^2}{(1-x)^2},$$

$$AdS: \qquad F(q^2) = \int_0^\infty dz \Phi(z) J(q^2, z) \Phi(z) \qquad .$$

where $\Phi(z)$ correspond to modes that represent hadrons and $J(q^2, z)$ represent to electromagnetic current.
Notice that if we consider $z = \zeta$, and if we can write $J(q^2, z)$ as

$$J(q^2, \zeta) = \int_0^1 dx\ f(x) J_0(\zeta q \sqrt{\frac{1-x}{x}}),$$

Relationship between Mesonic Wave Function and AdS Modes.

$$|\psi(x, \zeta)|^2 = A \frac{1}{\zeta} x(1-x) f(x) |\Phi(\zeta)|^2,$$

⋆ Dual modes to Mesons.

$$S_\Phi = \frac{(-1)^J}{2} \int d^d x dz \sqrt{g} e^{-\phi(z)} \left(\partial_N \Phi_J \partial^N \Phi^J - \mu_J^2(z) \Phi_J \Phi^J \right),$$

$$ds^2 = \left(\frac{R}{z}\right)^2 (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2) \quad \text{and} \quad \phi(z) = \kappa^2 z^2$$

$$\left[-\frac{d^2}{dz^2} + U_J(z) \right] \Phi_{nJ}(z) = M_{nJ}^2 \Phi_{nJ}(z)$$

where $U_J(z)$ is a effective potential given by

$$U_J(z) = \kappa^4 z^2 + \frac{4a_J^2 - 1}{4z^2} + 2\kappa^2 \left(b_J - 1 \right)$$

$$a_J = \frac{1}{2} \sqrt{(d - 2J)^2 + 4(\mu_J R)^2} \ , \ b_J = \frac{1}{2} \left(g_J R^2 + d - 2J \right) \text{ and } g_J R^2 = 4(J - 1)$$

Wave Function in momentum space

$$\psi_{q_1 \bar{q}_2}(x, k) = \frac{4\pi A}{\kappa \sqrt{x(1-x)}} \exp\left(-\frac{k^2}{2\kappa_1^2 x(1-x)} \right).$$

⋆ **Model Extension.**

• Introduction of massive quarks:

$$\frac{k^2}{x(1-x)} \to K = \frac{k^2}{x(1-x)} + m_{12}^2 \quad , \quad m_{12}^2 = \frac{m_1^2}{x} + \frac{m_2^2}{1-x}.$$

Equivalent to the following change of the kinetic term in the Schrödinger EOM:

$$-\frac{d^2}{d\zeta^2} \to -\frac{d^2}{d\zeta^2} + m_{12}^2.$$

Wave Function in momentum space

$$\psi_{q_1\bar{q}_2}(x,k) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} f(x, m_1, m_2) \exp\left(-\frac{k^2}{2\kappa_1^2 x(1-x)}\right). \quad \textit{with} \quad f(x, m_1, m_2) = A\, f(x) e^{-\frac{m_{12}^2}{2\lambda^2}}$$

• Extending the effective potential $U \to U + U_{\mathrm{C}} + U_{\mathrm{HF}}$, where U_{C} and U_{HF} are the contributions of the color Coulomb and hyperfine (HF) potentials.

$$M_{nJ}^2 = 4\kappa^2\left(n + \frac{L+J}{2}\right) + \int_0^1 dx\left(\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right) f^2(x, m_1, m_2) - \frac{64\alpha_s^2 m_1 m_2}{9\,(n+L+1)^2} + \frac{32\pi\alpha_s}{9}\,\frac{\beta_S\, v}{\mu_{12}}.$$

⋆ **Choice of parameters.**

- Constituent quark masses:
  $m = 420 MeV,\ m_s = 570 MeV,\ m_c = 1.6 GeV,\ m_b = 4.8 GeV$
- Dilaton Parameter: $\kappa = 550 MeV$
- Hiperfine Parameter: $v = 10^{-4} GeV^3$
- Constant Coupling with IR freezing.

$$\alpha_s(\mu^2) = \frac{12\pi}{33 - 2N_f} \frac{1}{ln(\frac{\mu^2 + M_B^2}{\Lambda^2})}$$

  With $M_B = 854 MeV$ and $\Lambda = 420 MeV$.

- $\lambda_{qq} = 0.63 GeV,\ \lambda_{qs} = 1.20 GeV,\ \lambda_{ss} = 1.68 GeV,$
  $\lambda_{qc} = 2.50 GeV,\ \lambda_{sc} = 3.00 GeV,\ \lambda_{qb} = 3.89 GeV,$
  $\lambda_{sb} = 4.18 GeV,\ \lambda_{cc} = 4.04 GeV,\ \lambda_{cb} = 4.82 GeV,\ \lambda_{bb} = 6.77 GeV.$

⋆ **Results.**

Masses of light mesons

Meson	n	L	S	Mass [MeV]			
π	0	0,1,2,3	0	140	1355	1777	2099
π	0,1,2,3	0	0	140	1355	1777	2099
K	0	0,1,2,3	0	496	1505	1901	2207
η	0,1,2,3	0	0	544	1552	1946	2248
$f_0[\bar{n}n]$	0,1,2,3	1	1	1114	1600	1952	2244
$f_0[\bar{s}s]$	0,1,2,3	1	1	1304	1762	2093	2372
$a_0(980)$	0,1,2,3	1	1	1114	1600	1952	2372
$\rho(770)$	0,1,2,3	0	1	804	1565	1942	2240
$\rho(770)$	0	0,1,2,3	1	804	1565	1942	2240
$\omega(782)$	0,1,2,3	0	1	804	1565	1942	2240
$\omega(782)$	0	0,1,2,3	1	804	1565	1942	2240
$\phi(1020)$	0,1,2,3	0	1	1019	1818	2170	2447
$a_1(1260)$	0,1,2,3	1	1	1358	1779	2101	2375

Masses of heavy-light mesons

Meson	J^P	n	L	S	Mass [MeV]			
$D(1870)$	0^-	0	0,1,2,3	0	1857	2435	2696	2905
$D^*(2010)$	1^-	0	0,1,2,3	1	2015	2547	2797	3000
$D_s(1969)$	0^-	0	0,1,2,3	0	1963	2621	2883	3085
$D_s^*(2107)$	1^-	0	0,1,2,3	1	2113	2725	2977	3173
$B(5279)$	0^-	0	0,1,2,3	0	5279	5791	5964	6089
$B^*(5325)$	1^-	0	0,1,2,3	1	5336	5843	6015	6139
$B_s(5366)$	0^-	0	0,1,2,3	0	5360	5941	6124	6250
$B_s^*(5413)$	1^-	0	0,1,2,3	1	5416	5992	6173	6298

Masses of heavy quarkonia $c\bar{c}$, $b\bar{b}$ and $c\bar{b}$

Meson	J^P	n	L	S	Mass [MeV]			
$\eta_c(2986)$	0^-	0,1,2,3	0	0	2997	3717	3962	4141
$\psi(3097))$	1^-	0,1,2,3	0	1	3097	3798	4038	4213
$\chi_{c0}(3414)$	0^+	0,1,2,3	1	1	3635	3885	4067	4226
$\chi_{c1}(3510)$	1^+	0,1,2,3	1	1	3718	3963	4141	4297
$\chi_{c2}(3555)$	2^+	0,1,2,3	1	1	3798	4038	4213	4367
$\eta_b(9300)$	0^-	0,1,2,3	0	0	9428	10190	10372	10473
$\Upsilon(9460)$	1^-	0,1,2,3	0	1	9460	10219	10401	10502
$\chi_{b0}(9860)$	0^+	0,1,2,3	1	1	10160	10343	10444	10521
$\chi_{b1}(9893)$	1^+	0,1,2,3	1	1	10190	10372	10473	10550
$\chi_{b2}(9912)$	2^+	0,1,2,3	1	1	10219	10401	10502	10579
$B_c(6276)$	0^-	0,1,2,3	0	0	6276	6911	7092	7209

Decay constants f_P (MeV) of pseudoscalar mesons

Meson	Data	Our
π^-	$130.4 \pm 0.03 \pm 0.2$	131
K^-	$156.1 \pm 0.2 \pm 0.8$	155
D^+	206.7 ± 8.9	167
D_s^+	257.5 ± 6.1	170
B^-	193 ± 11	139
B_s^0	$253 \pm 8 \pm 7$	144
B_c	$489 \pm 5 \pm 3$	159

Decay constants f_V (MeV) of vector mesons with open and hidden flavor

Meson	Data	Our	Meson	Data	Our
ρ^+	210.5 ± 0.6	170	ρ^0	154.7 ± 0.7	120
D^*	$245 \pm 20^{+3}_{-2}$	167	ω	45.8 ± 0.8	40
D_s^*	$272 \pm 16^{+3}_{-20}$	170	ϕ	76 ± 1.2	66
B^*	$196 \pm 24^{+39}_{-2}$	139	J/ψ	277.6 ± 4	116
B_s^*	$229 \pm 20^{+41}_{-16}$	144	$\Upsilon(1s)$	238.5 ± 5.5	56

Generalized Parton Distributions in a Holographical Model [3]

[3] A. V, Ivan Schmidt, Thomas Gutsche, Valery E. Lyubovitskij, Phys.Rev.D83:036001,2011

◇ General Ideas.

⋆ Electromagnetic form factors and GPDs.

$$F_1^p(t) = \int_0^1 dx \left(\frac{2}{3} H_v^u(x,t) - \frac{1}{3} H_v^d(x,t)\right)$$

$$F_1^n(t) = \int_0^1 dx \left(\frac{2}{3} H_v^d(x,t) - \frac{1}{3} H_v^u(x,t)\right)$$

$$F_2^p(t) = \int_0^1 dx \left(\frac{2}{3} E_v^u(x,t) - \frac{1}{3} E_v^d(x,t)\right)$$

$$F_2^n(t) = \int_0^1 dx \left(\frac{2}{3} E_v^d(x,t) - \frac{1}{3} E_v^u(x,t)\right)$$

⋆ Form Factors in AdS / QCD.

(Z. Abidin and C. E. Carlson, Phys. Rev. D**79**, 115003 (2009))

$$F_1^p(t) = C_1(Q^2) + \eta_p C_2(Q^2)\ ,\ \ F_1^n(t) = \eta_p C_3(Q^2)$$

$$F_2^p(t) = \eta_n C_2(Q^2) \text{ and } F_2^n(t) = \eta_n C_3(Q^2)$$

where

$$C_1(Q^2) = \int dz e^{-\Phi}(V(Q,z)/2z^3)(\psi_L^2(z) + \psi_R^2(z))$$

$$C_2(Q^2) = \int dz e^{-\Phi}(V(Q,z)/2z^2)(\psi_L^2(z) - \psi_R^2(z))$$

$$C_3(Q^2) = \int dz e^{-\Phi}(2m_N V(Q,z)/2z^3)(\psi_L^2(z)\psi_R^2(z))$$

$$V(Q,z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2} x^{\frac{Q^2}{4\kappa^2}} e^{-\frac{\kappa^2 z^2 x}{(1-x)}}$$

◇ Summary of Results

The GPDs obtained look like.

$$H_v^q(x,Q^2) = q(x)x^a \quad and \quad E_v^q(x,Q^2) = e(x)x^a,$$

where

$$a = Q^2/(4\kappa^2); \quad q(x) = \alpha^q\gamma_1 + \beta^q\gamma_2; \quad e(x) = \beta^q\gamma_3,$$

and

$$\alpha^u = 2\ ,\ \alpha^d = 1\ ,\ \beta^u = 2\eta_p + \eta_n\ ,\ \beta^d = \eta_p + 2\eta_n$$

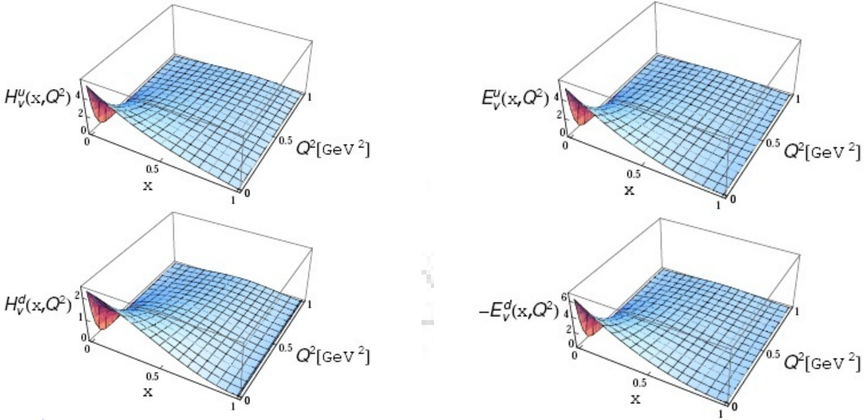
$$\gamma_1 = \frac{1}{2}(5 + 8x + 3x^2) \quad \gamma_1 = 1 - 10x + 21x^2 - 12x^3 \quad \gamma_1 = \frac{6m_N\sqrt{2}}{\kappa}(1-x)^2$$

⋆ **Parameters involved.**

$$\kappa = 350MeV\ ,\ \eta_p = 0.224\ ,\ \eta_n = -0.239$$

fixed to reproduce nucleon mass and anomalous magnetic moment of the nucleon.

◇ Nucleon GPDs Plots.



◇ Nucleon GPDs in impact space.

$$q(x, b_\perp) = \int \frac{d^2 k_\perp}{(2\pi)^2} H_q(x, k_\perp^2) e^{-ib_\perp k_\perp} \quad and \quad e^q(x, b_\perp) = \int \frac{d^2 k_\perp}{(2\pi)^2} E_q(x, k_\perp^2) e^{-ib_\perp k_\perp}$$

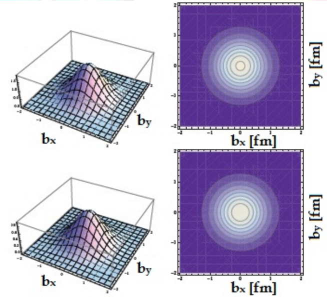
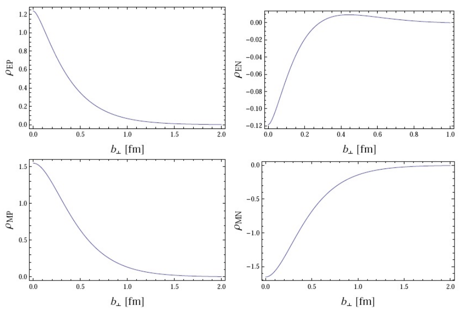


Figure: Plots for $q(x, b)$. The upper panels correspond to $u(x, b)$ and the lower to $d(x, b)$. Both cases are taken for $x = 0.1$.

◇ Parton charge and magnetization densities in transverse impact space.

$$\rho_E^N(b_\perp) = \sum_q e_q^N \int_0^1 dx\; q(x, b_\perp) \quad \textit{and} \quad \rho_M^N(b_\perp) = \sum_q e_q^N \int_0^1 dx\; e^q(x, b_\perp)$$



Conclusions

- In a phenomenological way we extended Brodsky and de Teramonds ideas to map AdS modes with mesonic wave functions with massive quarks, considering additional potentials.
- These ingredients let us describe several mesons.
- For other side we determined the nucleon GPDs using a similar procedure used in some applications of light front holography.
- The nucleon GPDs obtained have an exponential form, as in several phenomenological approaches.
- As in hadronic physics the mesonic wave function and GPDs are very important, we can see that Gauge / Gravity can be considered as a useful tool in some QCD applications.

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That's all Folks !